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Department of Engineering Science
Aerospace Engineering and Nuclear Engineering

FACULTY OF ENGINEERING AND APPLIED SCIENCES

State University of New York at Buffalo

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LONG WAVE TRAPPING BY A LINEAR RIDGE TOPOGRAPHY

by

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and

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LONG WAVE TRAPPING BY A LINEAR RIDGE TOPOGRAPHY

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ABSTRACT

Long waves are affected by bottom topography and under certain conditions may be trapped along topographical contours which then act as wave guides transmitting wave energy for great distances with little loss. This study examines waves trapped along a submerged ridge described by straight parallel bottom contours which in cross section are composed of constant slope segments bounded on either side by constant depth segments. Solutions are found for time harmonic waves periodic in the along ridge direction and of exponential decay behavior normal to the ridge over the constant depth segments. Over the linearly varying topography describing the ridge, the solution is in terms of two Kummer (or Whittaker) functions. For a given geometry, a dispersion equation is obtained relating the wave frequency to the along ridge wave number for trapped waves. A constant Coriolis parameter is included, but primary interest is on class I (high frequency) waves. A comparison of cutoff frequencies predicted for this piecewise continuous ridge and those for a segmented constant depth ridge is made, and the appropriate scaling factors between the two results are discussed.

INTRODUCTION

The study of topographically trapped waves appears to have begun with Stokes' description of edge waves on a linear topography of straight parallel bottom contours, Stokes (1846). While this solution was originally felt to be of only theoretical interest, observations during the last few decades have given new physical significance to these forms of waves. This study shall examine the influence of one dimensional (parallel contours) bottom topography on the trapping and guiding of ocean wave energy. In particular, linearized long wave theory, for a homogeneous perfect fluid, with rotation is used over topographies which consist of constant and linear depth cross sectional variations.
Before further detail on this study, a review of some background material is in order. LeBlond and Mysak (1978) provide an important overview of the area of ocean waves; more specific results are given below.
Eckert (1951) showed that, for shallow water theory, the Stokes solution was just the lowest mode of an infinity of modes and Ursell (1952) obtained these results generalized to three dimensional linearized water wave theory leading to a mixed spectrum of discrete and continuous eigenfrequencies.
Reid (1958) showed that the earth's rotation would have a modifying effect on these class I or inertio-gravitational waves (which can propagate in both directions along a N-S coast albeit with differing phase speeds for non-zero rotation) and would also allow for a class II or quasi-geostrophic wave which can propagate in only one direction and can not exist in the absence of rotation.

The differences between class I (high frequency) and class II (low frequency) shallow water waves has lead to two roughly distinct classes of literature--those primarily concerned with relatively nearshore phenomena, e.g., tsunami periods and shorter (under 1 hour), see Munk, Snodgrass and Carrier (1956), and those concerned with oceanographic scale phenomena, e.g., periods of the order of days. Clearly, these interests overlap but a distinction can be made on the basis of the wave frequency, ω, versus the Coriolis frequency, f. If $\omega \gg f$, it is the "edge" wave or class I wave which is usually of primary interest while if $\omega \ll f$, it is the "quasi-geostrophic" or class II wave. This study is directed at the former; the second class of waves will be discussed in a later paper.

It is clear from the various formulations of these problems that waves may be trapped over completely submerged topography, as shown by Hidaka (1976)--see Meyer (1971) for a review of resonance of unbounded water bodies, Shen, Meyer and Keller (1968) for a geometrical optics approach and Buchwald (1968) for a solution of the piecewise constant depth case and a discussion of the general nature of such problems as solutions to Sturm-Liouville equations. One aim of the present study is to see how well the piecewise constant depth solution represents solutions for gently sloping bottom topographies by comparison to another analytical solution for piecewise linear depths. The solution to this linear topography was noted by Shaw (1974) and Guza and Davis (1974) in terms of Kummer functions, but no numerical results were given until Shaw (1977), due to an apparent lack of tables and/or subroutines for these functions.

While the question of resonances is difficult enough for the trapped wave problem, the prior question of how energy arrives to be trapped must also be considered. For trapped waves, the energy may be supplied by nonlinear effects, local atmospheric forcing or even local seismic disturbances since a linearized, unforced theory only provides for the persistence of such trapped motions once begun. However, it must be pointed out that total wave trapping is exceptional; Longuet-Higgins (1969) has shown that curved topographies lead to "leaky" trapped modes. Rather than negating the importance of trapped wave studies, these results enhance them, since this provides a simple mechanism for these cases by which incident wave energy can be fed into the resonating system. Alternatively, for a ridge of finite length, wave energy may enter the "system" at the ends, exciting a resonance along the ridge as well as across the ridge.
This mechanism may be particularly appropriate at locations where a long ridge terminates at a coastline, e.g. the Chattham rise off of the east coast of New Zealand, Heath (1979).

FORMULATION

The problems considered here are concerned with wave energy trapping by a topography with either a one dimensional piecewise discontinuous cross section composed of constant depth segments or a piecewise continuous cross section composed of linear depth segments. Although rotation of an f plane will be included, emphasis will be on first class surface gravity waves on a homogeneous fluid where Coriolis effects are a modifying rather than a fundamental influence.

The vertically integrated equations of motion in a coordinate system with contours parallel to the y axis are

$$\begin{aligned}
&\text{a)}\quad \partial u/\partial t - fv = -g\ \partial\zeta/\partial x \\
&\text{b)}\quad \partial v/\partial t + fu = -g\ \partial\zeta/\partial y \qquad (1) \\
&\text{c)}\quad \partial[Hu]/\partial x + \partial[Hv]/\partial y = -\partial\zeta/\partial t
\end{aligned}$$

with water depth H and Coriolis parameter f.

Assuming time harmonic behavior, $\exp(-i\omega t)$, and periodic along-contour behavior, $\exp(iky)$, allows these equations to be solved separately for the horizontal velocities u, v and free surface elevation, ζ.

$$\begin{aligned}
&\text{a)}\quad u = [-i\omega g\ \partial\zeta/\partial x + ikg\zeta]/[\omega^2 - f^2] \\
&\text{b)}\quad v = [\omega gk\zeta + fg\ \partial\zeta/\partial x]/[\omega^2 - f^2] \qquad (2) \\
&\text{c)}\quad H(x)\frac{d^2\zeta}{dx^2} + \frac{dH(x)}{dx}\frac{d\zeta}{dx} + \left[\frac{\omega^2-f^2}{g} - \frac{fk}{\omega}\frac{dH}{dx} - Hk^2\right]\zeta = 0
\end{aligned}$$

These equations may be non-dimensionalized with respect to some reference depth H_R and length L, i.e., using

$$\bar{\zeta} = \zeta/H_R,\ \bar{h} = H/H_R$$

$$(\bar{x},\bar{y}) = (x,y)/L,\ \bar{k} = kL$$

$$(\bar{u},\bar{v}) = (u,v)\left(\omega L/gH_R\right)$$

$$\bar{f} = f/\omega,\ \bar{\omega} = \omega L/\left(gH_R\right)^{1/2}$$

equations (2) become

$$\bar{u} = i[\bar{f}\bar{k}\bar{\zeta} - \partial\bar{\zeta}/\partial\bar{x}]/[1 - \bar{f}^2]$$

$$\bar{v} = [\bar{k}\bar{\zeta} + \bar{f}\partial\bar{\zeta}/\partial\bar{x}]/[1 - \bar{f}^2]$$

$$\bar{h}(\bar{x})\frac{d^2\bar{\zeta}}{d\bar{x}^2} + \frac{d\bar{h}(\bar{x})}{d\bar{x}}\frac{d\bar{\zeta}}{d\bar{x}} + \left[\bar{\omega}^2(1 - \bar{f}^2) - \bar{k}\bar{f}\frac{d\bar{h}}{d\bar{x}} - \bar{h}\bar{k}^2\right]\bar{\zeta} = 0$$

which will have different solutions for different topographies, $\bar{h}(\bar{x})$. The simplest case arises for constant depth, e.g., H_o (not necessarily the reference depth) such that $\bar{h}_o = H_o/H_R$,

$$\bar{h}_o \frac{d^2\bar{\zeta}}{d\bar{x}^2} + [\bar{\omega}^2(1 - \bar{f}^2) - \bar{h}_o\bar{k}^2]\bar{\zeta} = 0$$

leading to

$$\bar{\zeta} = A \exp[+\lambda\bar{x}] + B \exp[-\lambda\bar{x}]$$

where λ is defined by the indicial equation

$$\lambda^2 = \bar{k}^2 - \bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_o$$

For values of $\bar{h}_o$ and $\bar{f}$ such that $\bar{k}^2 > \bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_o$, λ is real and the solution for $\bar{\zeta}$ is exponential in $\bar{x}$ while for $\bar{k}^2 < \bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_o$, λ is imaginary and the solution is sinusoidal in $\bar{x}$.

If a linear depth is considered, $\bar{h}(\bar{x}) = [\delta + \gamma\bar{x}]$, this equation reduces to

$$[\delta + \gamma\bar{x}] \frac{d^2\bar{\zeta}}{d\bar{x}^2} + \gamma \frac{d\bar{\zeta}}{d\bar{x}} + [\bar{\omega}^2(1 - \bar{f}^2) - \bar{k}\bar{f}\gamma - (\delta + \gamma\bar{x})\bar{k}^2]\bar{\zeta} = 0$$

Introduce $z = \delta + \gamma\bar{x}$; this equation is then

$$z \frac{d^2\bar{\zeta}}{dz^2} + \frac{d\bar{\zeta}}{dz} + \left[\frac{\bar{\omega}^2(1 - \bar{f}^2) - \bar{k}\bar{f}\gamma}{\gamma^2} - \frac{\bar{k}^2}{\gamma^2} z\right]\bar{\zeta} = 0$$

which is reducible to the confluent hypergeometric equation, e.g., Erdelyi, et al. (1953). Solutions to this equation are of the form, with $\xi > 0$:

$$\bar{\zeta} = \exp[-\xi/2]\left\{c_1 M(a,1,\xi) + c_2 U(a,1,\xi)\right\}$$

where $\xi = -2\bar{k}z/\gamma$, $a = (1 - \bar{f})/2 + \bar{\omega}^2(1 - \bar{f}^2)/2\gamma\bar{k}$, $(\gamma \neq 0)$ and M and U are Kummer Functions, with arbitrary coefficients c_1 and c_2.

For $\xi < 0$, the Kummer transformation leads to an alternate form:

$$\bar{\zeta} = \exp[+\xi/2]\left\{c_1' \, M(a',1,-\xi) + c_2' \, U(a',1,-\xi)\right\}$$

where $a' = 1 - a = \frac{1}{2} + \frac{\bar{f}}{2} - \frac{\bar{\omega}^2(1-\bar{f}^2)}{2\gamma\bar{k}}$

A program for Kummer functions of both kinds is given by Shaw and Neu (1977) in an internal SUNY-B report and is available from the authors.

STEP RIDGE TOPOGRAPHY - DISCONTINUOUS, CONSTANT DEPTH CROSS SECTION

This problem was discussed by Buchwald (1968) and forms a test case against which the later continuous depth case may be examined. Since constant depth, discontinuous (piecewise continuous) topographies are much simpler to use than any other topography, it is of great practical interest to know, analytically, how well they may represent a gradually varying topography. The basic comparison will be done for the dispersion equation, in particular for the low frequency cutoff frequencies and other significant properties.

Consider a cross section defined by three regions of constant depth

Region I: $-\infty < x < -B$; $H(x) = H_1$

Region II: $-B < x < +B$; $H(x) = H_2$

Region III: $+B < x < +\infty$; $H(x) = H_3$

where coordinates are chosen such that $H_3 \geq H_1$.

Using B and H_1 as reference lengths for horizontal and vertical distances respectively, the solution in Region I is

$$\bar{\zeta}_I = A_1 \exp\left[\lambda_1 \bar{x}\right] \tag{5}$$

where $\lambda_1 = [\bar{k}^2 - \bar{\omega}^2(1 - \bar{f}^2)]^{1/2}$ and the portion of the solution which diverges as $\bar{x} \to -\infty$ is surpressed. In Region II, an oscillatory behavior is anticipated, i.e.

$$\bar{\zeta}_{II} = A_2 \cos\left(\lambda_2 \bar{x}\right) + B_2 \sin\left(\lambda_2 \bar{x}\right) \tag{6}$$

where $\lambda_2 = \left[\bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_2 - \bar{k}^2\right]^{1/2}$. Finally, in Region III we anticipate another exponentially decaying solution

$$\bar{\zeta}_{III} = A_3 \exp\left[-\lambda_3 \bar{x}\right] \tag{7}$$

where $\lambda_3 = \left[\bar{k}^2 - \bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_3\right]^{1/2}$.

Clearly $\bar{k}$ is bounded by $\bar{\omega}\left[(1 - \bar{f}^2)/\bar{h}_2\right]^{1/2}$ from above and by $\bar{\omega}[1 - \bar{f}^2]^{1/2}$ from below.

Continuity of surface elevation and mass flow requires

$$\bar{\zeta}_I\ (\bar{x} = -1) = \bar{\zeta}_{II}\ (\bar{x} = -1)$$

$$\bar{u}_I\ (\bar{x} = -1) = \bar{h}_2\ \bar{u}_{II}\ (\bar{x} = -1)$$

$$\bar{\zeta}_{II}(\bar{x} = +1) = \bar{\zeta}_{III}(\bar{x} = +1)$$

$$\bar{h}_2\ \bar{u}_{II}(\bar{x} = +1) = \bar{h}_3\ \bar{u}_{III}(\bar{x} = +1)$$

leading to a system of 4 homogeneous linear algebraic equations on $\left[A_1, A_2, B_2, A_3\right]$. The determinant of the coefficient matrix of this system must vanish in order to have a non-trivial solution; this defines the dispersion equation. For simplicity, consider the symmetric case, $H_1 = H_3$, without rotation, $f = 0$. The dispersion equation then reduces to

$$2\bar{h}_2\ \lambda_1\lambda_2 \cos 2\lambda_2 = \left(\bar{h}_2^2\ \lambda_2^2 - \lambda_1^2\right)\sin 2\lambda_2$$

Along the line $\bar{\omega} = \bar{k}$ which forms a lower bound on the allowable values of $\bar{k}$ for trapped waves, $\lambda_1 = 0$ and the dispersion equation reduces to

$$\sin 2\lambda_2 = 0, \quad \lambda_2 = n\pi/2; \quad n = 0,1,2,3. . .$$

These are the cutoff points, corresponding to

$$\bar{\omega}_n = (n\pi/2)\left|\bar{h}_2/\left(1-\bar{h}_2\right)\right|^{1/2}$$

$$\bar{k}_n = (n\pi/2)\left|\bar{h}_2/\left(1-\bar{h}_2\right)\right|^{1/2}$$

The shape of the free surface separates into symmetric and antisymmetric modes. For even n, B_2 is zero and $A_1 = A_3 = \cos(n\pi/2)A_2$, i.e. these are the symmetric modes, while for odd n, A_2 is zero and $A_1 = -A_3 = -\sin(n\pi/2)A_2$, i.e. the antisymmetric modes.

LINEAR RIDGE TOPOGRAPHY - CONTINUOUS LINEAR CROSS SECTION

Consider a ridge composed of a piecewise continuous, linear cross section defined by

Region I: $-\infty < x < -A$; $H(x) = H_1$

Region II: $-A < x < 0$; $H(x) = H_2 - \left(H_1 - H_2\right)x/A$

Region III: $0 < x < B$; $H(x) = H_2 + \left(H_3 - H_2\right)x/B$

Region IV: $B < x < \infty$; $H(x) = H_3$

as shown in Figure 1. A suitable reference depth is $H_R = H_1$ and reference length $L = A$.

Then in Region I, $\bar{h}_1 = 1$ and

$$\bar{\zeta}_I = A_1 \exp\left[\lambda_1 \bar{x}\right] + B_1 \exp\left[-\lambda_1 \bar{x}\right]$$

where

$$\lambda_1 = [\bar{k}^2 - \bar{\omega}^2(1 - \bar{f}^2)]^{1/2}$$

Clearly, B_1 must be set equal to zero and $\bar{k}^2 > \bar{\omega}^2(1 - \bar{f}^2)$ to have an exponentially decaying wave as $\bar{x} \to -\infty$.

Similarly in Region IV, $\bar{h} = H_3/H_1 = \bar{h}_3$ and

$$\bar{\zeta}_{IV} = A_4 \exp\left[-\lambda_4 \bar{x}\right] + B_4 \exp\left[+\lambda_4 \bar{x}\right]$$

where

$$\lambda_4 = \left[\bar{k}^2 - \bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_3\right]^{1/2}$$

Again we require a decaying solution as $\bar{x} \to +\infty$, and thus $B_4 = 0$ and $\bar{k}^2 > \bar{\omega}^2(1 - \bar{f}^2)/\bar{h}_3$.

Next, in Region II, $\bar{h}(x) = \bar{h}_2 - \left(1 - \bar{h}_2\right)\bar{x}$ such that $\delta = \bar{h}_2 < 1$ and $\gamma = -\left(1 - \bar{h}_2\right) < 0$. Then

$$\bar{\zeta}_{II} = \exp\left[-\xi_2/2\right]\left\{A_2 M\left(a_2, 1, +\xi_2\right) + B_2 U\left(a_2, 1, +\xi_2\right)\right\}$$

with $\xi_2 = +2\bar{k}\left[\left(\bar{h}_2/\left(1-\bar{h}_2\right)\right) - \bar{x}\right]$ and $a_2 = \frac{1}{2} - \frac{\bar{f}}{2} \quad -\bar{\omega}^2(1-\bar{f}^2)/2\left(1-\bar{h}_2\right)\bar{k}$.

Finally, in Region III, $\bar{h}(\bar{x}) = \bar{h}_2 + \left(\bar{h}_3 - \bar{h}_2\right)(A/B)\bar{x}$ such that $\delta = \bar{h}_2 < 1$ and $\gamma = \left(\bar{h}_3 - \bar{h}_2\right)(A/B) > 0$.

Then $\bar{\zeta}_{III} = \exp\left[+\xi_3/2\right]\left\{A_3 M\left(a_3', 1, -\xi_3\right) + B_3 U\left(a_3', 1, -\xi_3\right)\right\}$

with $\xi_3 = -2\bar{k}\left[\bar{h}_2\bar{b}/\left(\bar{h}_3 - \bar{h}_2\right) + \bar{x}\right]$ and $a_3' = (1+\bar{f})/2 - \bar{\omega}^2(1-\bar{f}^2)\bar{b}/2\bar{k}\left(\bar{h}_3 - \bar{h}_2\right)$.

Continuity of free surface height and mass flux at $\bar{x} = -1$, $\bar{x} = B/A = \bar{b}$ requires

$$
\begin{aligned}
\bar{\zeta}_I\,(-1) &= \bar{\zeta}_{II}(-1)\\
\bar{u}_I\,(-1) &= \bar{u}_{II}(-1)\\
\bar{\zeta}_{II}\,(0) &= \bar{\zeta}_{III}(0)\\
\bar{u}_{II}\,(0) &= \bar{u}_{III}(0)\\
\bar{\zeta}_{III}(\bar{b}) &= \bar{\zeta}_{IV}\,(\bar{b})\\
\bar{u}_{III}(\bar{b}) &= \bar{u}_{IV}\,(\bar{b})
\end{aligned}
$$

or
$$
\left[C_{ij}\right]\begin{bmatrix} A_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \\ A_4 \end{bmatrix} = 0
$$

Using $\mu_1 = \bar{k}/1-\bar{h}_2$, $\mu_2 = \bar{h}_2\mu_1$, $\mu_3 = \bar{k}\bar{h}_2\bar{b}/\bar{h}_3-\bar{h}_2$, $\mu_4 = \bar{k}\bar{h}_3\bar{b}/\bar{h}_3-\bar{h}_2$ and $\alpha_1 \equiv \left(a_2, 1, 2\mu_1\right)$; $\alpha_2 \equiv \left(a_2, 1, 2\mu_2\right)$; $\beta_1 \equiv \left(a_3', 1, 2\mu_3\right)$; $\beta_2 \equiv \left(a_3', 1, 2\mu_4\right)$ and $M' \equiv dM/d\xi$, $U' \equiv dU/d\xi$, the non-zero values of C_{ij} are

$$
\begin{aligned}
C_{11} &= -\exp\left(-\lambda_1\right)\\
C_{12} &= +\exp\left(-\mu_1\right)M\left(\alpha_1\right)\\
C_{13} &= +\exp\left(-\mu_1\right)U\left(\alpha_1\right)
\end{aligned}
$$

$$C_{21} = -\lambda_1 \exp\left(-\lambda_1\right)$$
$$C_{22} = \bar{k}\ \exp\left(-\mu_1\right)\left[M\left(\alpha_1\right) - 2M'\left(\alpha_1\right)\right]$$
$$C_{23} = \bar{k}\ \exp\left(-\mu_1\right)\left[U\left(\alpha_1\right) - 2U'\left(\alpha_1\right)\right]$$
$$C_{32} = \exp\left(-\mu_2\right) M\left(\alpha_2\right)$$
$$C_{33} = \exp\left(-\mu_2\right) U\left(\alpha_2\right)$$
$$C_{34} = -\exp\left(-\mu_3\right) M\left(\beta_1\right)$$
$$C_{35} = -\exp\left(-\mu_3\right) U\left(\beta_1\right)$$
$$C_{42} = \bar{k}\ \exp\left(-\mu_2\right)\left[M\left(\alpha_2\right) - 2M'\left(\alpha_2\right)\right]$$
$$C_{43} = \bar{k}\ \exp\left(-\mu_2\right)\left[U\left(\alpha_2\right) - 2U'\left(\alpha_2\right)\right]$$
$$C_{44} = +\bar{k}\ \exp\left(-\mu_3\right)\left[M\left(\beta_1\right) - 2M'\left(\beta_1\right)\right]$$
$$C_{45} = +\bar{k}\ \exp\left(-\mu_3\right)\left[U\left(\beta_1\right) - 2U'\left(\beta_1\right)\right]$$
$$C_{54} = \exp\left(-\mu_4\right) M\left(\beta_2\right)$$
$$C_{55} = \exp\left(-\mu_4\right) U\left(\beta_2\right)$$
$$C_{56} = -\exp\left(-\lambda_4 \bar{b}\right)$$
$$C_{64} = \bar{k}\ \exp\left(-\mu_4\right)\left[M\left(\beta_2\right) - 2M'\left(\beta_2\right)\right]$$
$$C_{65} = \bar{k}\ \exp\left(-\mu_4\right)\left[U\left(\beta_2\right) - 2U'\left(\beta_2\right)\right]$$
$$C_{66} = -\lambda_4 \exp\left(-\lambda_4 \bar{b}\right)$$

Again, it is instructive to examine the case of no rotation, $f = 0$, and a symmetric topography, $A = B$ and $H_1 = H_3$. The cutoff points along the boundary curve $\bar{\omega} = \bar{k}$ may again be found using $\lambda_1 = \lambda_4 = 0$. For this case, $a_2 = a_3' = a$.

$$\bar{\zeta}_{I} = A_1$$
$$\bar{\zeta}_{IV} = A_4$$
$$\bar{\zeta}_{II} = \exp\left[-\bar{k}z/\left(1-\bar{h}_2\right)\right]\left\{A_2 M\left(a,1,2\bar{k}z/\left(1-\bar{h}_2\right)\right) + B_2 U\left(a,1,2\bar{k}z/\left(1-\bar{h}_2\right)\right)\right\}$$
$$\bar{\zeta}_{III} = \exp\left[-\bar{k}z/\left(1-\bar{h}_2\right)\right]\left\{A_3 M\left(a,1,2\bar{k}z/\left(1-\bar{h}_2\right)\right) + B_3 U\left(a,1,2\bar{k}z/\left(1-\bar{h}_2\right)\right)\right\}$$

where $z = \bar{h}_2 - \left(1-\bar{h}_2\right)\bar{x}$ in Region II and $\bar{h}_2 + \left(1-\bar{h}_2\right)\bar{x}$ in Region III. The continuity conditions require

$$\frac{d\bar{\zeta}_{II}}{d\bar{x}}\,(\bar{x} = -1) = 0$$

$$\frac{d\bar{\zeta}_{III}}{d\bar{x}}\,(\bar{x} = +1) = 0$$

$$\bar{\zeta}_{II}\,(\bar{x} = 0) = \bar{\zeta}_{III}\,(\bar{x} = 0)$$

$$\frac{d\bar{\zeta}_{II}}{d\bar{x}}\,(\bar{x} = 0) = \frac{d\bar{\zeta}_{III}}{d\bar{x}}\,(\bar{x} = 0)$$

or
$$\begin{bmatrix} M(\alpha) & U(\alpha) & -M(\alpha) & -U(\alpha) \\ M(\alpha)-2M'(\alpha) & U(\alpha)-2U'(\alpha) & M(\alpha)-2M'(\alpha) & U(\alpha)-2U'(\alpha) \\ M(\beta)-2M'(\beta) & U(\beta)-2U'(\beta) & 0 & 0 \\ 0 & 0 & M(\beta)-2M'(\beta) & U(\beta)-2U'(\beta) \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \\ A_3 \\ B_3 \end{bmatrix} = 0$$

where $\alpha \equiv \alpha_2$ and $\beta \equiv \beta_2$.

The determinant of these coefficients, which must vanish for a nontrivial solution, may be reduced to a product

$$\begin{vmatrix} M(\alpha)-2M'(\alpha) & U(\alpha)-2U'(\alpha) \\ M(\beta)-2M'(\beta) & U(\beta)-2U'(\beta) \end{vmatrix} * \begin{vmatrix} M(\alpha) & U(\alpha) \\ M(\beta)-2M'(\beta) & U(\beta)-2U'(\beta) \end{vmatrix} = 0 \qquad (13)$$

which corresponds to symmetric modes $(d\bar{\xi}/d\bar{x} = 0$ at $\bar{x} = 0,1)$ and antisymmetric modes $(\bar{\xi} = 0$ at $\bar{x} = 0$, $d\bar{\xi}/d\bar{x} = 0$ at $\bar{x} = 1)$ respectively.

Numerical results are shown in Figures 2 - 5 for the symmetric case $H_1 = 3$ Km, $H_2 = 1$ Km, $H_3 = 3$ Km and $A = B = 60$ Km and a slightly asymmetric case, $H_1 = 3$ Km, $H_2 = 1$ Km, $H_3 = 4.85$ Km and $A = 60$ Km, $B = 78$ Km, both without rotation, $f = 0$. Figure 2 shows the symmetric case dispersion equation followed by Figure 3 which show the first two mode shapes. Note that both symmetric and antisymmetric modes result. Figure 4 shows the asymmetric case

dispersion equation followed by Figure 5 which shows the first two mode shapes; note the slight shift in mode shape caused by the asymmetry.

COMPARISON OF CUTOFF FREQUENCIES

The cutoff frequencies for a discontinuous, segmented constant depth ridge defined by depth H_1 for $x < -B_C$, H_2 for $-B_C < x < B_C$ and H_3 for $x > +B_C$ are given by

$$\bar{\omega}_n = (n\pi/2)\left[\bar{h}_2/\left(1-\bar{h}_2\right)\right]^{1/2}$$

where the depth H_1 and half ridge width B_C are again used as reference lengths, e.g. Buchwald (1968), Shaw and Neu (1979). Corresponding cutoff frequencies may be found for the linear ridge for various values of $\bar{h}_2$; these are shown in Table 1:

TABLE 1 - LINEAR RIDGE CUTOFF FREQUENCIES, $\bar{\omega}_n\left(\bar{h}_2\right)$

$\bar{h}_2$ \ n	1	2	3	4	5	6
0.1	1.395	3.220	4.570	6.189	7.595	9.166
0.2	1.791	3.796	5.502	7.360	9.094	10.920
0.3	2.171	4.394	6.424	8.541	10.583	12.678
0.4	2.580	5.064	7.438	9.849	12.223	14.620
0.5	3.053	5.862	8.635	11.400	14.160	
0.6	3.645	6.881	10.150	13.368		
0.7	4.456	8.297	12.249			
0.8	5.800	10.567				

The ratio of these cutoff frequencies to those for the constant depth case varies with $\bar{h}_2$ and slightly with n and is shown in Figure 6. Here, $\bar{\omega}_n$ (constant depth)/$\bar{\omega}_n$ (linear depth) is shown as a function of $\bar{h}_2$ with a range indicated at each $\bar{h}_2$ representing the variation with n. For a wide range of values of $\bar{h}_2$, the ratio is approximately 0.5. This implies that a linear ridge of total width $2B_L$ and a constant depth segment ridge of one half of this width, $B_C = 0.5\ B_L$, would lead to approximately the same cutoff frequencies. This corresponds to maintaining the same total cross sectional area of the ridge and is physically appealing as a useful approximation allowing a complicated problem (linear ridge) to be replaced by a much simpler problem (constant depth ridge).

Of course, this example does not include rotation and is only for the symmetric ridge. Furthermore, there are other features of the dispersion equation which should also be matched, e.g. the minimum in the group velocity noted by Buchwald (1968). However the initial point (cutoff frequency) and the asymptote $\left(\bar{\omega}^2 = \bar{h}_2\bar{k}^2\right)$ are well matched by this substitution. This is particularly gratifying in view of the drastic difference in bottom slopes which arise in these two different cases.

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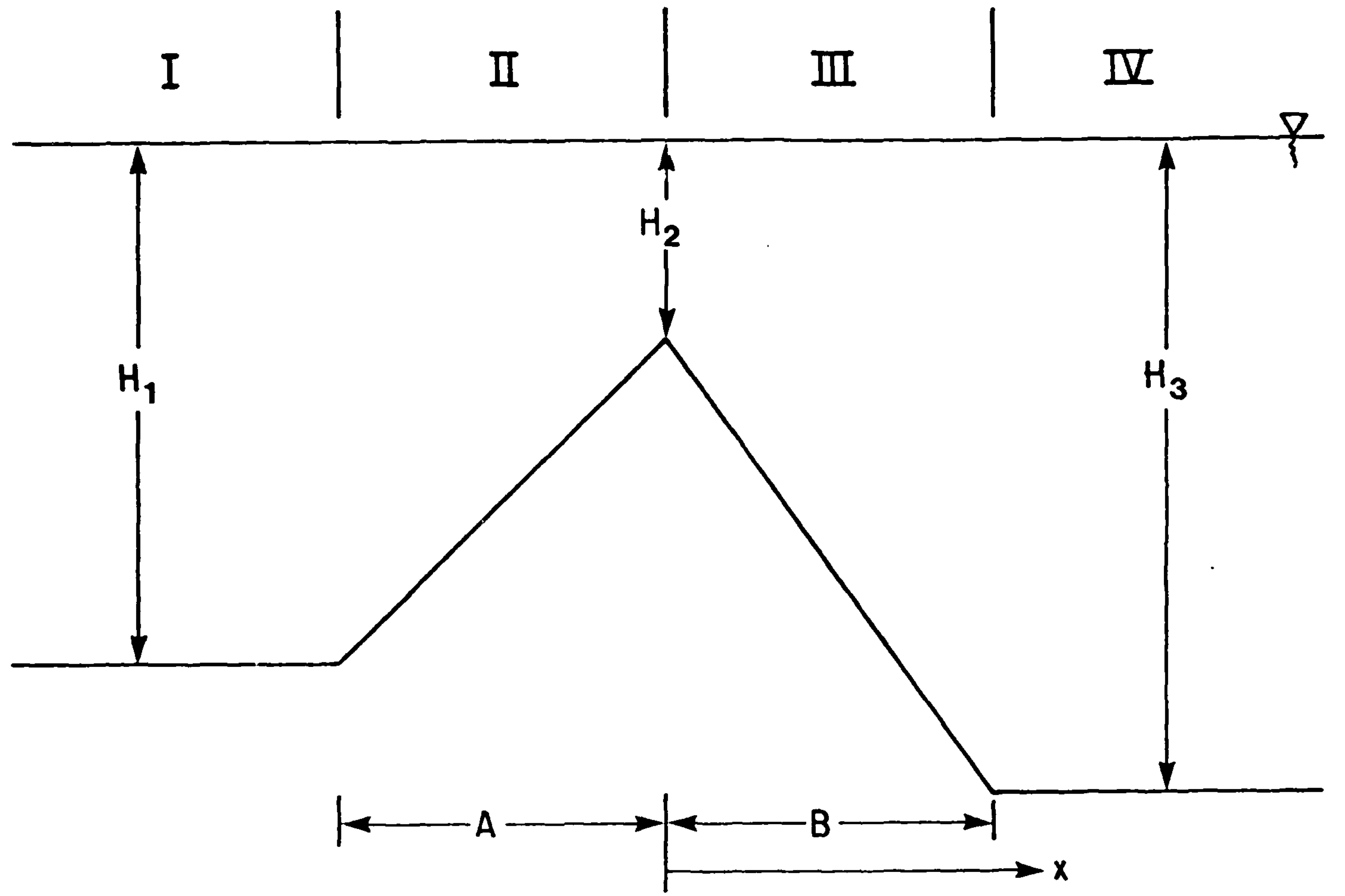
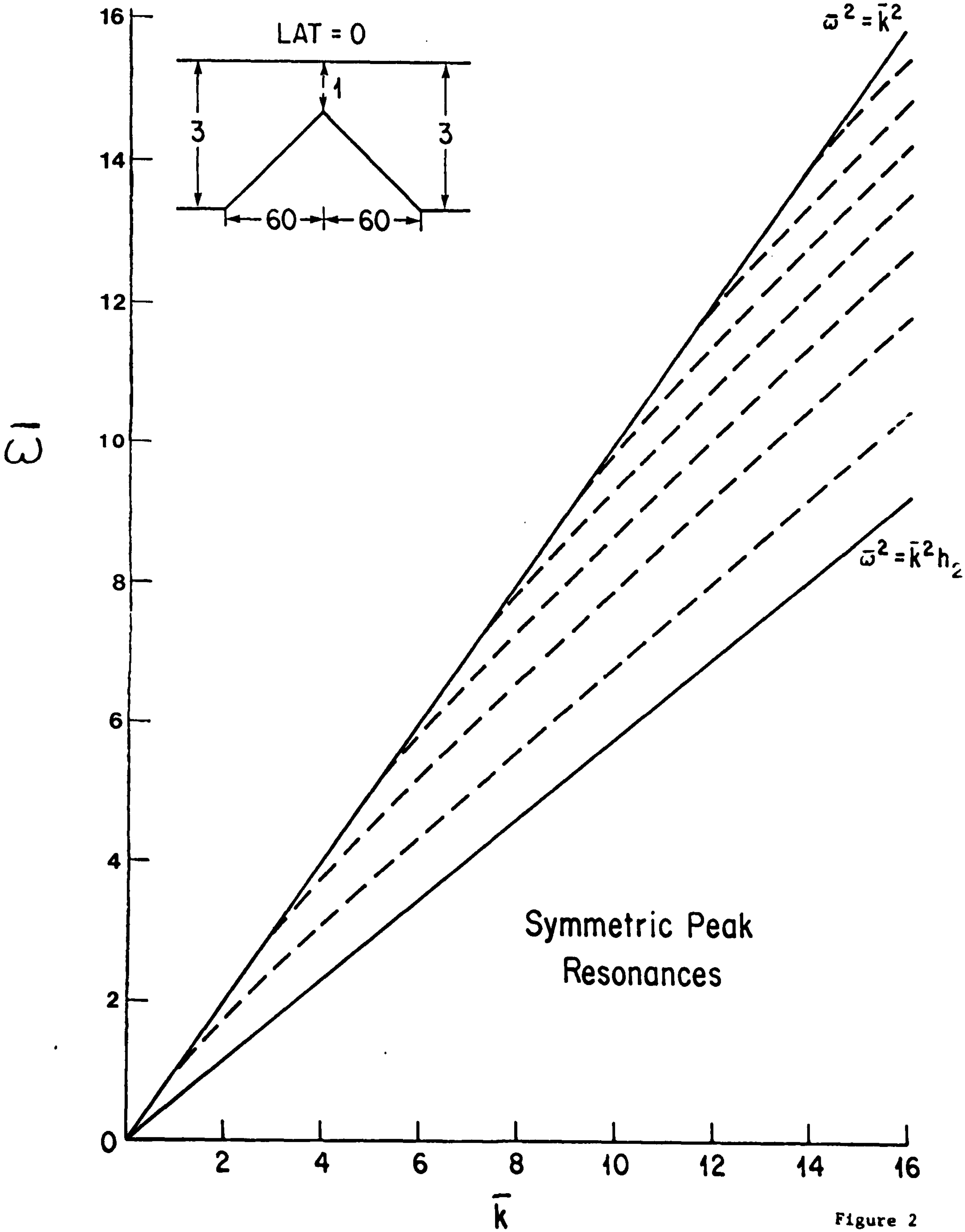
TABLE OF FIGURES



Linear Segment Ridge Approximation

Figure 1



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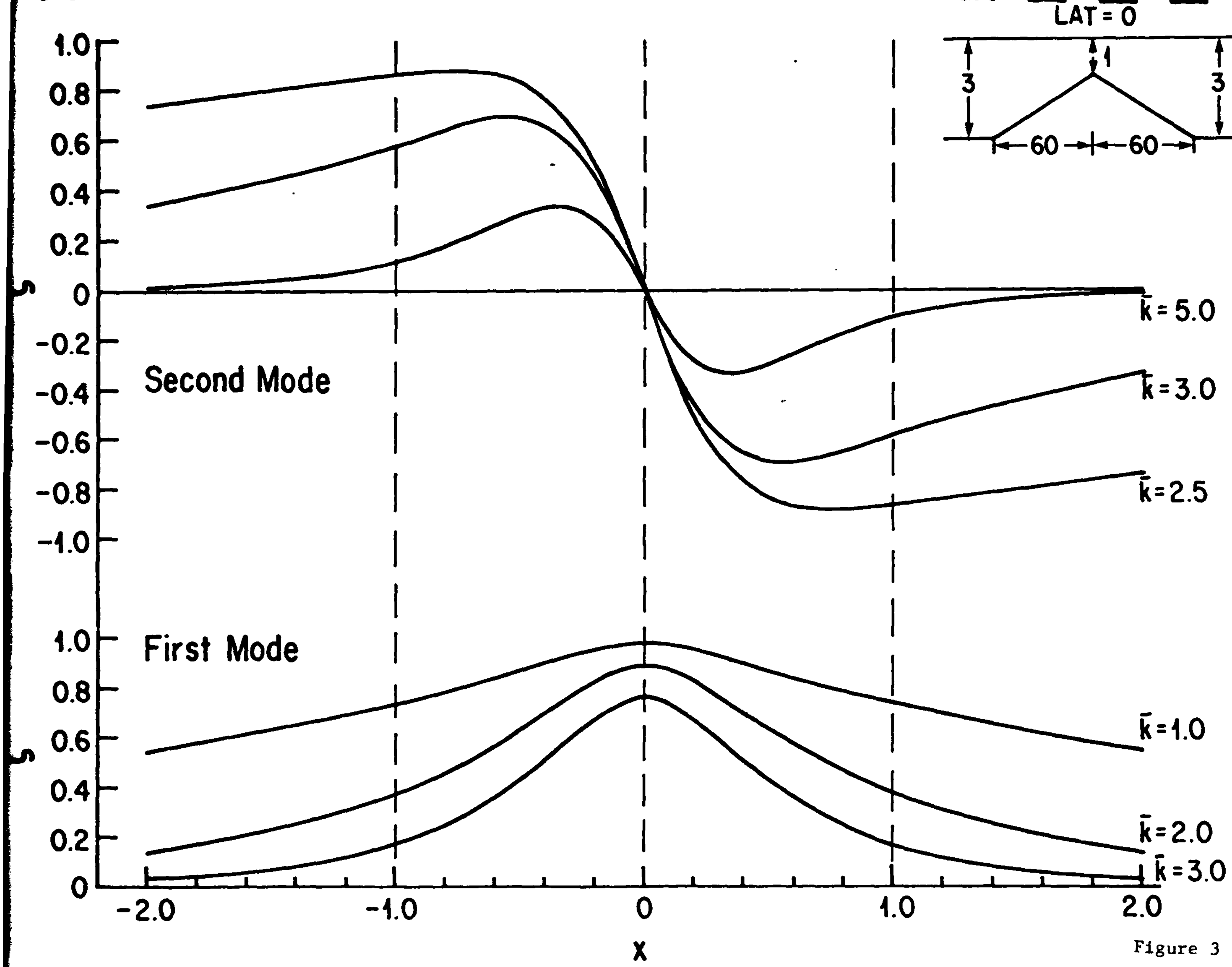
3
3
60
60
16
14
12
10
8
6
4
2
0
2
4
6
8
10
12
14
16
$\bar{\omega}^2 = \bar{k}^2$
$\bar{\omega}^2 = \bar{k}^2 h_2$
$\bar{\omega}$
$\bar{k}$
Symmetric Peak
Resonances

Figure 2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0.8
0.6
0.4
0.2
0
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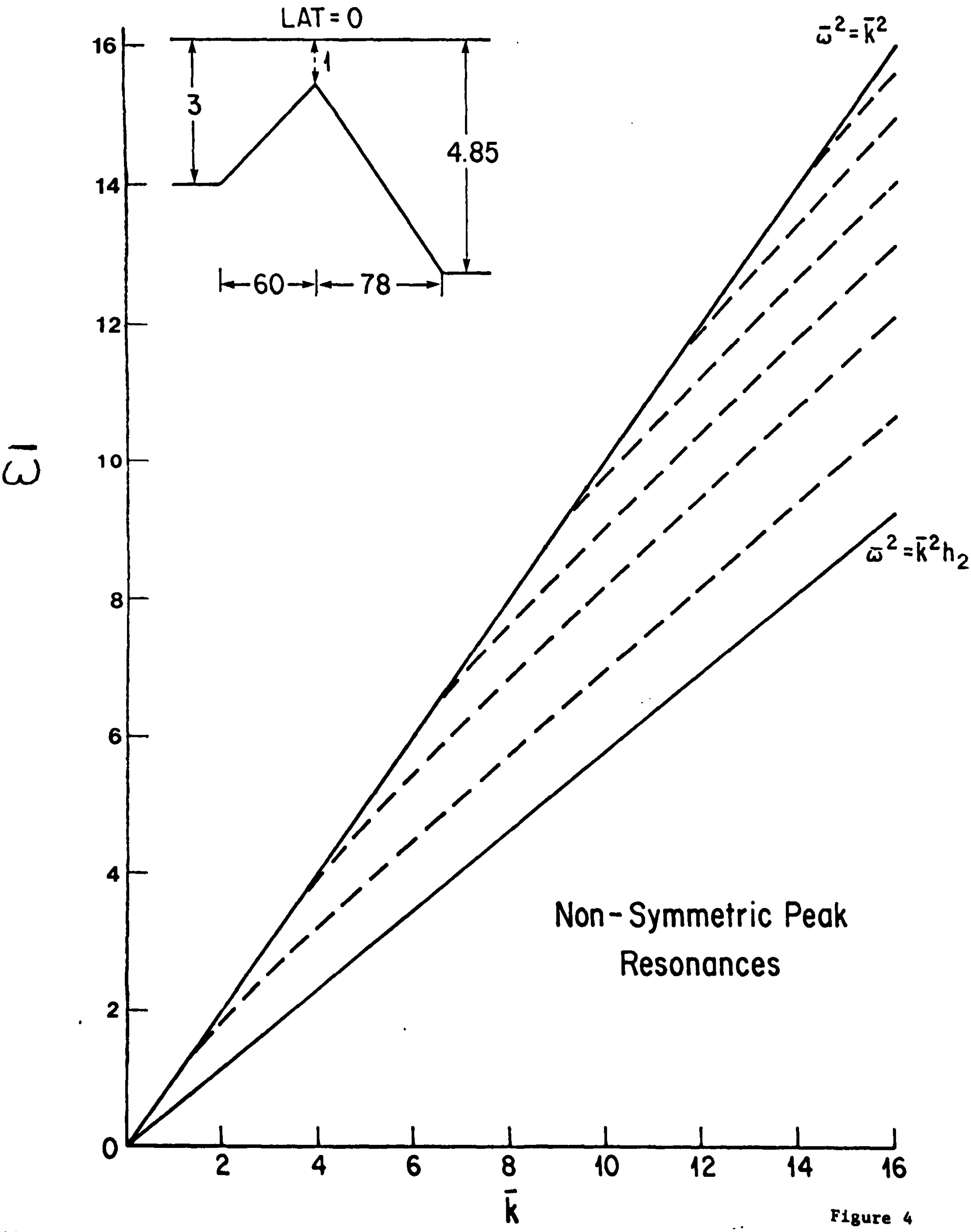
-0.4
-0.6
-0.8
-1.0
Second Mode
First Mode
$\bar{k}$ = 5.0
$\bar{k}$ = 3.0
$\bar{k}$ = 2.5
$\bar{k}$ = 1.0
$\bar{k}$ = 2.0
$\bar{k}$ = 3.0
-2.0
-1.0
0
1.0
2.0
X

Figure 3



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78
16
14
12
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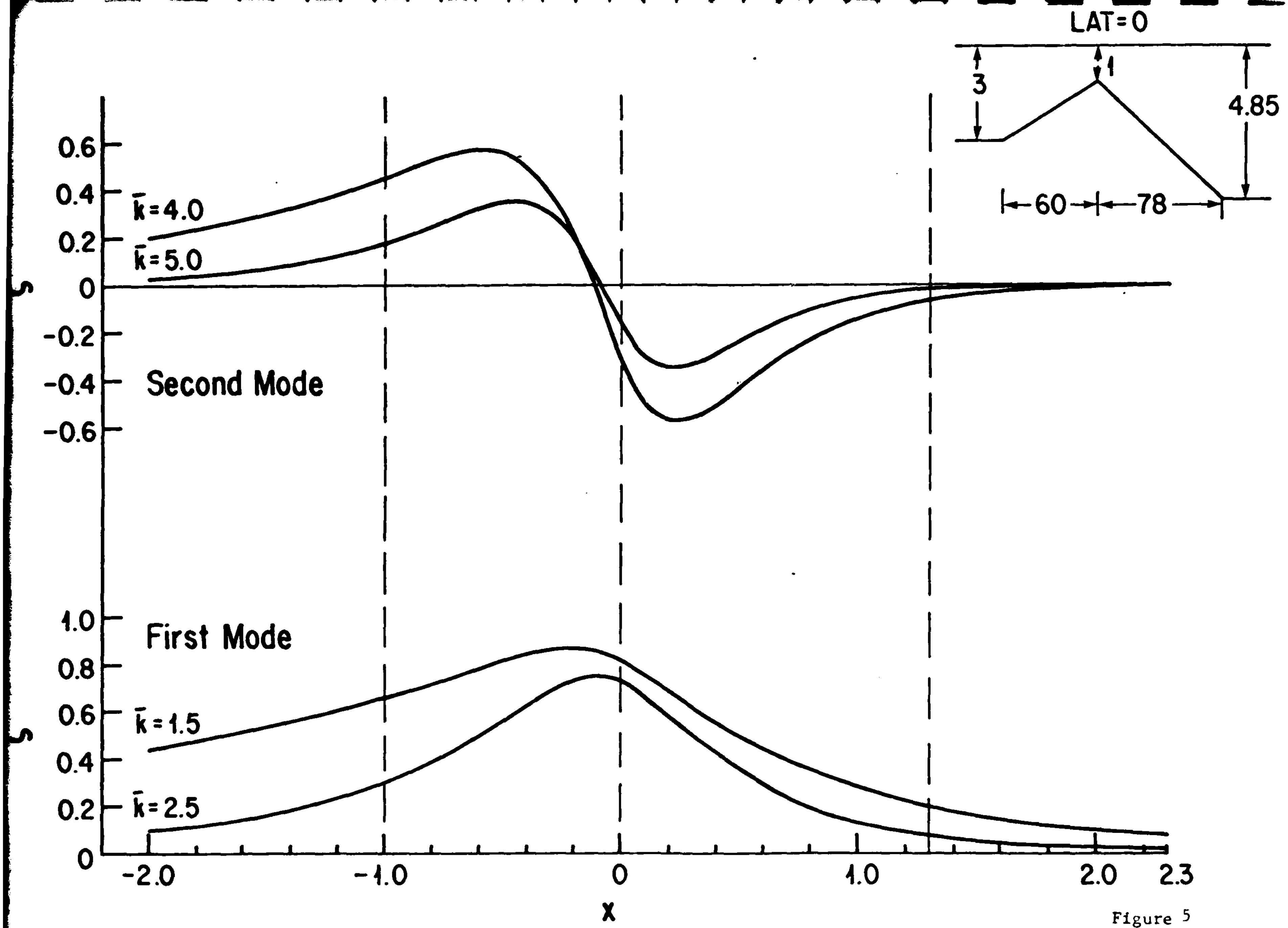
8
6
4
2
0
2
4
6
8
10
12
14
16
$\bar{\omega}$
$\bar{k}$
$\bar{\omega}^2 = \bar{k}^2$
$\bar{\omega}^2 = \bar{k}^2 h_2$
Non-Symmetric Peak
Resonances

Figure 4



LAT=0
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1
4.85
60
78
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
ς
$\bar{k}$=4.0
$\bar{k}$=5.0
Second Mode
1.0
0.8
0.6
0.4
0.2
0
ς
First Mode
$\bar{k}$=1.5
$\bar{k}$=2.5
-2.0
-1.0
0
1.0
2.0
2.3
X

Figure 5

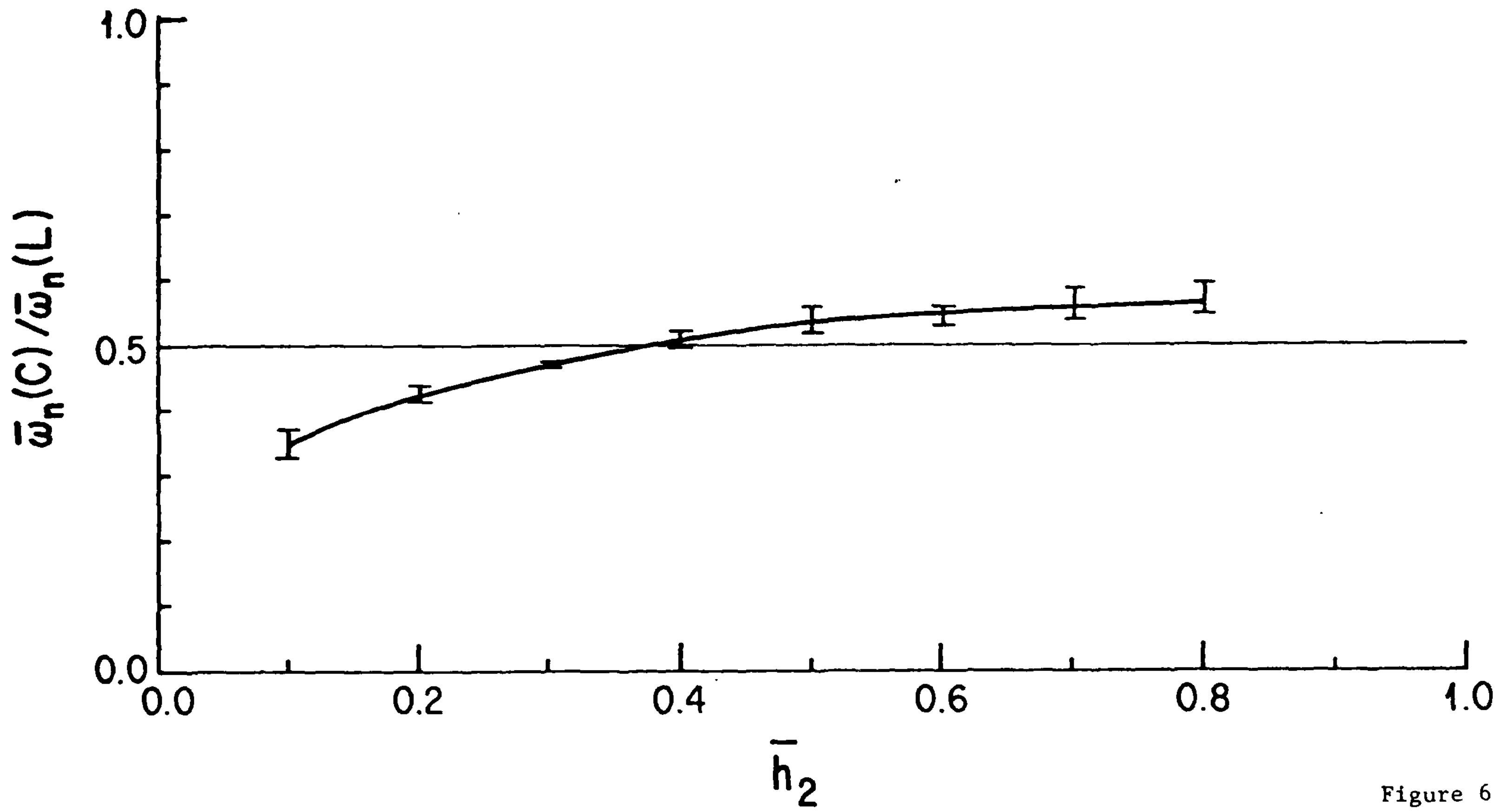

Figure 6

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Edge Waves

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Long waves are affected by bottom topography and under certain conditions may be trapped along topographical contours which then act as wave guides transmitting wave energy for great distances with little loss. This study examines waves trapped along a submerged ridge described by straight parallel bottom contours which in cross section are composed of constant slope segments bounded on either side by constant depth segments. Solutions are found for time harmonic waves periodic in the along ridge direction and of exponential

(turn over)

decay behavior normal to the ridge over the constant depth segments. Over the linearly varying topography describing the ridge, the solution is in terms of two Kummer (or Whittaker) functions. For a given geometry, a dispersion equation is obtained relating the wave frequency to the along ridge wave number for trapped waves. A constant Coriolis parameter is included, but primary interest is on class I (high frequency) waves. A comparison of cutoff frequencies predicted for this piecewise continuous ridge and those for a segmented constant depth ridge is made, and the appropriate scaling factors between the two results are discussed.

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